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Kalman Filter Techniques

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Abstract

This paper answers several questions of centralized Kalman-Filters in multi-sensor fusion, fault detection and isolation in sensors, optimal control in linear-quadratic Gaussian problem, an algorithm in fuzzy based approach to adaptive Kalman-Filtering additionally in multi-state multi-sensor fusion. Generally, Kalman-Filters comprise a number of types and topologies depending on use and computing complexity of applied processors. State estimation provided by a Kalman-Filter is crucial in this thesis. Kalman-Filter performs optimal estimation of an unknown system state through filters behavior. This thesis supposes some models of promising linear Kalman-Filter simulated beyond MATLAB and Simulink program especially utilized in the fields of steering-controls or navigations, etc.

Index Terms — Kalman-*Filter*, multi-sensorfusion, Fuzzy-Logic, Gaussian, Optimal estimate

1. Introduction

In this paper, we dedicate the effort to introduce Kalman filter - KF techniques with 2 models of conventional Kalman filter, CoKF, mainly. Although there is no difference between centralized Kalman filter CKF and CoKF, we like to show the CoKF as an estimator structure in single-sensor systems First of all, we will assume a mathematical model of a plant defined by equations of discrete system dynamics. To get the equations of the optimum estimator, i.e., the KF, suppose that the plant of system dynamics are designed by the (possibly time-varying) general model of linear finite-dimensional stochastic system, see below; [1], [2].

$$\mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n) + \mathbf{B}w(n) \qquad (1\text{-}1)$$

$$y_V(n) = \mathbf{C}\mathbf{x}(n) + v(n), \quad n \geq n_0 \qquad (1\text{-}2$$

2. Model 1 of Kalman filter

In this part, investigates a timing diagram of KF In order to get a control program flow with applied equations in Table 2-1 below. This will be also introduced briefly in next model. The model refers to [1 - 2]. The table deals with two programs, i.e. initial program and main iterative program. The Initial time n_0 is the formal time when processor does not process first sample but starts an initial program

Intional Program Initial Time n=0

$$\mathbf{P}(n|n-1) = \mathbf{B}\,\mathbf{Q_d}\,\mathbf{B}^T \text{ , where } \mathbf{Q_d} \text{ is defaulted } \mathbf{Q}(0) > \mathbf{0} \qquad (2\text{-}1)$$

$$\mathbf{x}(n|n-1) = \mathbf{0} \qquad (2\text{-}2)$$

$$y_e(n) = 0 \qquad (2\text{-}3)$$

Iteration Time n = 1,2,3,...

$$\mathbf{M}(n) = \mathbf{P}(n|n-1)\,\mathbf{C}^T \;/\; (\mathbf{C}\,\mathbf{P}(n|n-1)\,\mathbf{C}^T + R(n)) \qquad (2\text{-}4)$$

$$r(n) = y_V(n) - \mathbf{C}\,\mathbf{x}(n|n-1) \qquad (2\text{-}5)$$

$$\mathbf{x}(n|n) = \mathbf{x}(n|n-1) + \mathbf{M}(n)\, r(n) \qquad (2\text{-}6)$$

$$\mathbf{P}(n|n) = [\mathbf{I} - \mathbf{M}(n)\,\mathbf{C}]\,\mathbf{P}(n|n-1) \qquad (2\text{-}7)$$

$$y_e(n) = \mathbf{C}\,\mathbf{x}(n|n) \qquad (2\text{-}8)$$

$$\text{error cov} = \mathbf{C}\,\mathbf{P}(n|n)\,\mathbf{C}^T \qquad (2\text{-}9)$$

$$\mathbf{x}(n+1|n) = \mathbf{A}\,\mathbf{x}(n|n) + \mathbf{B}\,u(n) \qquad (2\text{-}10)$$

$$\mathbf{P}(n+1|n) = \mathbf{A}\,\mathbf{P}(n|n)\,\mathbf{A}^T + \mathbf{B}\,\mathbf{Q}(n)\,\mathbf{B}^T \qquad (2\text{-}11)$$

Table 2-1

The timing diagram of Table 2.1 can be described as follows
The innovation KF gain is computed in (2-10) with the usage of delayed matrix of error covariance time update, and the recent observation noise variance $\boldsymbol{R}(n)$ at the beginning of discrete time n. This refers to computation of state error covariance matrix, which indicates an accuracy of the state estimate. This calculation provides optimal innovation KF gain to minimize a KF cost function below

$$\boldsymbol{P}(n)= E\left([\boldsymbol{x}(n)- \boldsymbol{x}(n\mid n)][\boldsymbol{x}(n)- \boldsymbol{x}(n\mid n)]^T \right) \qquad (2\text{-}12)$$

3. Model 2 of Kalman filter

A timing diagram shown in Table 3.1 is modified model from Table 2-1 Both modelsare mathematically identical. A difference between these two models and their mathematical identity is measured because other expressions of error covariance update and innovation gain are used here. This way, we will slightly tend to DKF techniques in multi sensor fusion.

Initial Time n = 0

$\mathbf{P}(n|n\text{-}1) = \mathbf{B}\,\mathbf{Q_d}\,\mathbf{B}^T$ where $\mathbf{Q_d}$ is defaulted $\mathbf{Q}(0) > \mathbf{0}$ (3-1)

$\mathbf{x}(n|n\text{-}1) = \mathbf{0}$ (3-2)

$y_e(n)=0$ (3-3)

Iteration Time n = 1,2,3,…

Observation update :

Error Covariance update: $\mathbf{P}(n|n)^{-1} = \mathbf{P}(n|n\text{-}1)^{-1} + \mathbf{C}^T \mathbf{C} / R(n)$, (3-4)

Innovations Sequence (Residuals):
$r(n) = y_v(n) - \mathbf{C}\,\mathbf{x}(n|n\text{-}1)$ (3-5)

State Estimate Update:
$\mathbf{x}(n|n) = \mathbf{x}(n|n\text{-}1) + \mathbf{M}(n)\, r(n)$ (3-6)

Estimated Filter Output:
$y_e(n) = \mathbf{C}\,\mathbf{x}(n|n)$ (3-7)

Error Covariance:
error cov = $\mathbf{C}\,\mathbf{P}(n|n)\,\mathbf{C}^T$ (3-8)

Time update :

State Time Update:
$\mathbf{x}(n+1|n) = \mathbf{A}\,\mathbf{x}(n|n) + \mathbf{B}\,u(n)$ (3-9)

Error Covariance Time Update:
$\mathbf{P}(n+1|n) = \mathbf{A}\,\mathbf{P}(n|n)\,\mathbf{A}^T + \mathbf{B}\,\mathbf{Q}(n)\,\mathbf{B}^T$ (3-10)

Table 3-1

The state error covariance matrix becomes singular always at the beginning of simulation

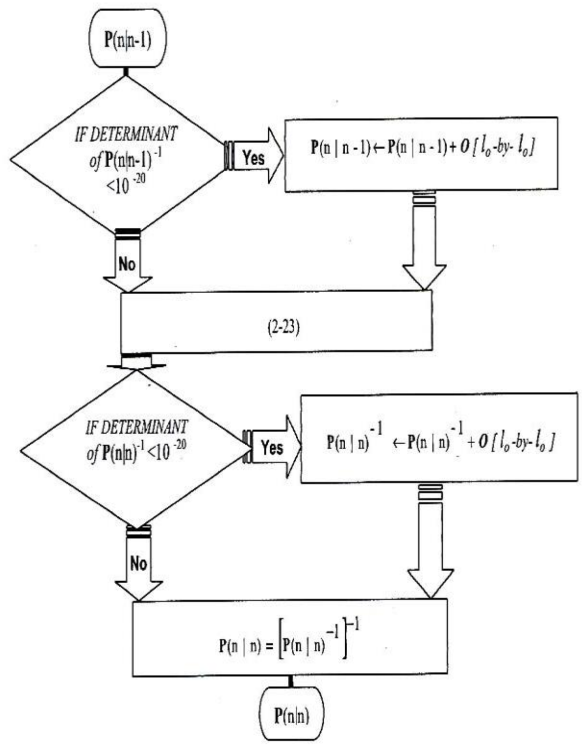

Fig 3-1

Elements $\boldsymbol{o}_{ii}$ refers to N(0,10power -14) and identity matrix $\boldsymbol{O}$ of noise N (0,10power-4) those elements $\boldsymbol{o}_{ii}$ are taken to absolute value $\boldsymbol{O} = [\boldsymbol{O}_{ii}] = \boldsymbol{o}_{ii}$, where i = 1,2, 3,..., l_O, and l_O is the order of state vector. The matrix is additionally summarized with state error covariance matrix when the singularity happens

4. Centralized Kalman filter Techniques

This deals with CKF technique and models. The models are built according to In centralized Kalman filtering, Signals of sensors are transferred through the communication network to the central processor to generate the optimal central estimate $\boldsymbol{x}(n|n)$. The all information is sent to the fusion centre, Figure 4-2, to yield $\boldsymbol{x}(n|n)$ and minimize state estimation error.

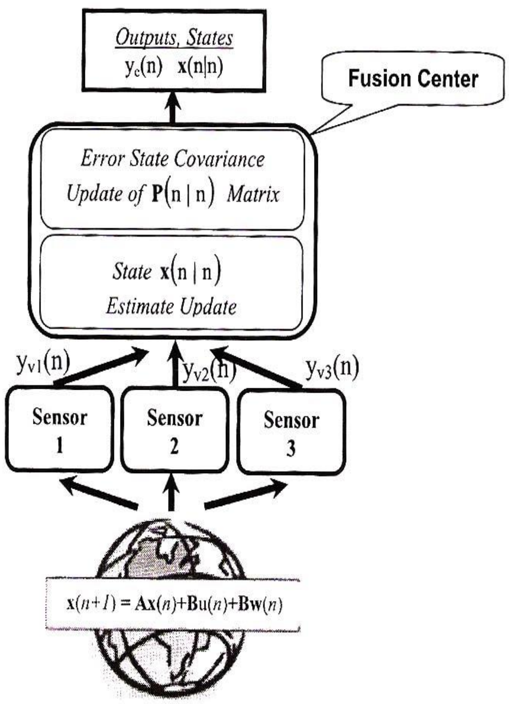

Fig 4-1 Centralized Kalman filter topology

5. Model 1 of centralized Kalman filter

Initial Program

Initial Time n = 0

Initial error covariance: $\mathbf{P}(n \mid n-1) = \mathbf{B}\,\mathbf{Qd}\,\mathbf{B}^{T}$,

$\mathbf{Q_d}$ is defaulted $\mathbf{Q}(0) > \mathbf{0}$ (5-1)

Initial condition on the state: $\mathbf{x}(n \mid n-1) = \mathbf{0}$ (5-2)

Initial condition on Filter Output: $y_{ei}(n) = 0$ (5-3)

Iteration Time n = 1,2,3,…

Observation update :

Innovation KF Gain:

$$\mathbf{M}\ (n) = \mathbf{P}(n \mid n-1)\ \mathbf{C}^{T}_{i} \left[\mathbf{C}_i\ \mathbf{P}(n \mid n-1)\,\mathbf{C}_i + R_i(n)\right]$$

(5-4)

Table 5-1

State Estimate Update

$\mathbf{x}(n \mid n) = \mathbf{x}(n \mid n-1) + 1/N\sum \mathbf{M}i=1\ ^{I}(n)\ r\ (n)$, $N \geq 1$ (5-5)

N0 means number of sensors

Error Covariance Update:

$$\mathbf{P}(n \mid n) = \left|\mathbf{I} - \ 1\ \ N^{oN}{}_{o}\sum \mathbf{M}i=1\ \ ^{i}(n)\,\mathbf{C}\ \ ^{i}\right|\mathbf{P}(n \mid n-1)$$

(5-6)

Estimated Filter Output:

$y_{ei}(n) = \mathbf{C}_i\mathbf{x}(n \mid n)$ (5-7)

Error Covariance:
error cov= $\mathbf{C}_i\mathbf{P}(n \mid n)$ **Ci power T** (5-8)

Time update :
State Time Update:
$\mathbf{x}(n+1 \mid n) = \mathbf{A}\,\mathbf{x}(n \mid n) + \mathbf{B}\,u(n)$ (5-9)

Error Covariance Time Update:
$\mathbf{P}(n+1 \mid n) = \mathbf{A}\,\mathbf{P}(n \mid n)\ \mathbf{A}^{T} + \mathbf{B}\,\mathbf{Q}(n)\,\mathbf{B}^{T}$ (5-10)

Table 5-2 Timing diagram of Model 1 of CKF.

Update in digital computing. To get over this problem we present following method capable to avoid the singularity Time updated covariance simulation. This inconvenience causes wrong state estimate matrix becomes singular always at beginning of simulation and in a term of middle time during whole simulation shown by a flowchart in Figure 5.1

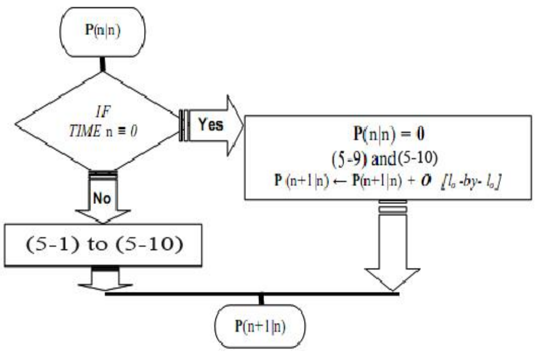

Fig 5-1 Treatment of the DKF error covariance time update calculation

6. Results

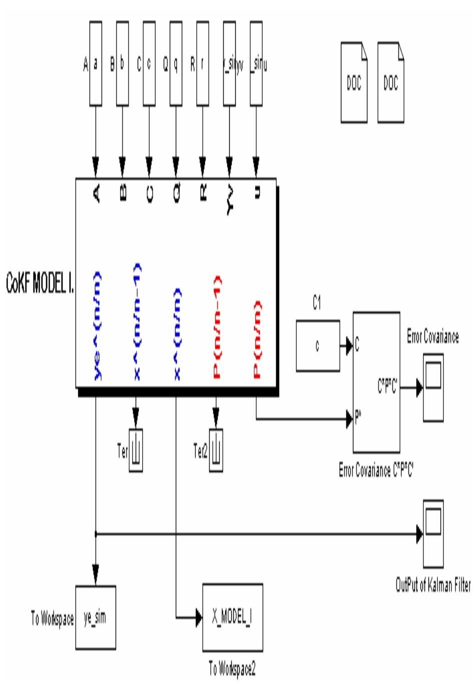

Fig 6-1Simulink Block of Model 1 of KF

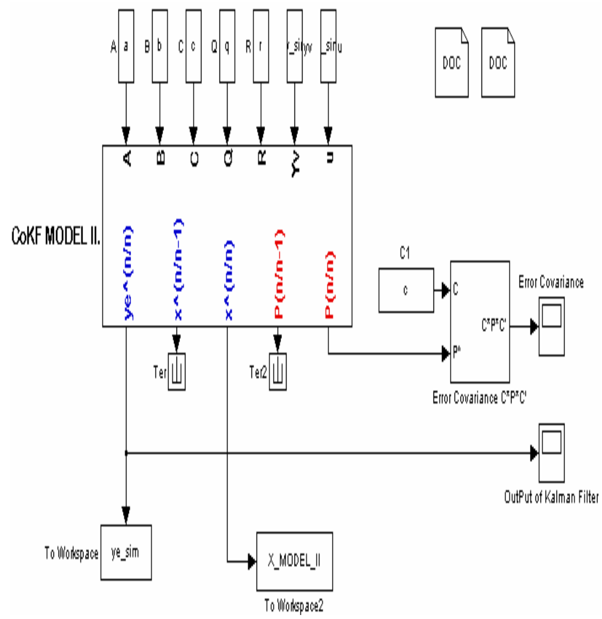

Fig 6-2 Model 2 of KF in Simulink MATLAB Program

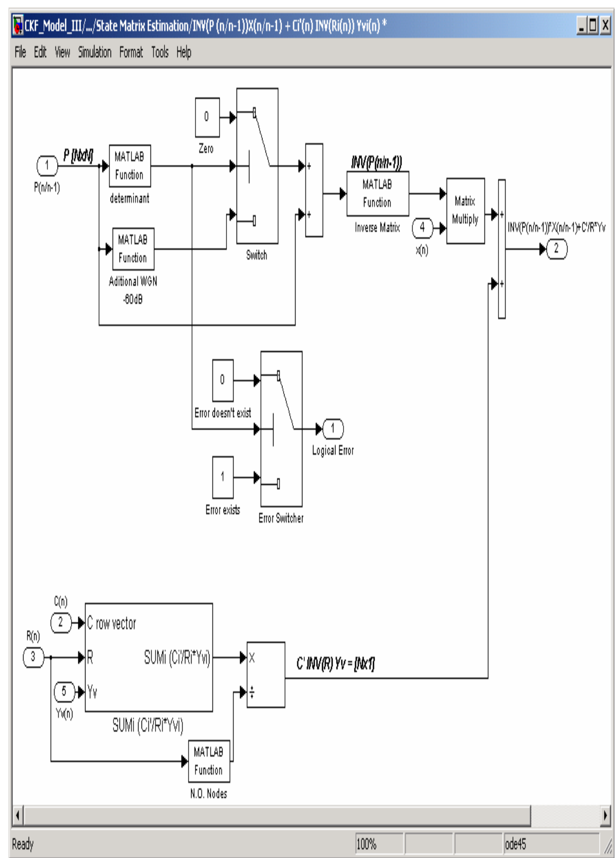

***Fig 6-3*Model 3 of CKF in Simulink MATLAB Program**

7. References

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8. Conclusion

This manuscript is composed of two main parts as well as Kalman Filter techniques, MATALAB simulation and Tests. The first part employs Kalman-Filter technique with one sensor, The second part centralized Kalman- Filter technique.. Centralized Kalman-Filter with many sensors so-called CKF. Then the followings are associated with experiments in problematic of bias, broken node, drift effect on state estimation in DKF, algorithm of fault detection and isolation in sensors, algorithm and exercising on adaptive fuzzy logic centralized Kalman-Filter, LQR and LQG.